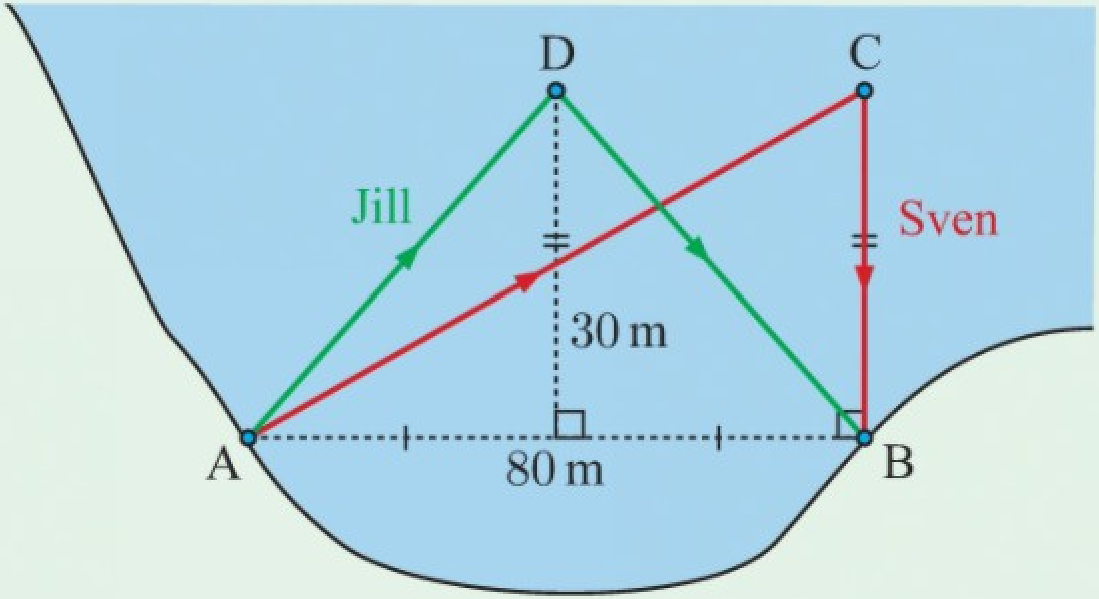
Pythagoras' theorem

OPENING PROBLEM

Sven has challenged Jill to a swimming race in a lake. Both swimmers will swim from A to B, but each needs to touch a buoy along the way. Sven thinks he is a very good swimmer, so he will swim to buoy C on his way to B. Jill will swim to buoy D, as shown.



Things to think about:

- **a** How far will Sven swim?
- **b** How far will Jill swim?
- **c** How much further will Sven swim than Jill?

A

PYTHAGORAS’ THEOREM

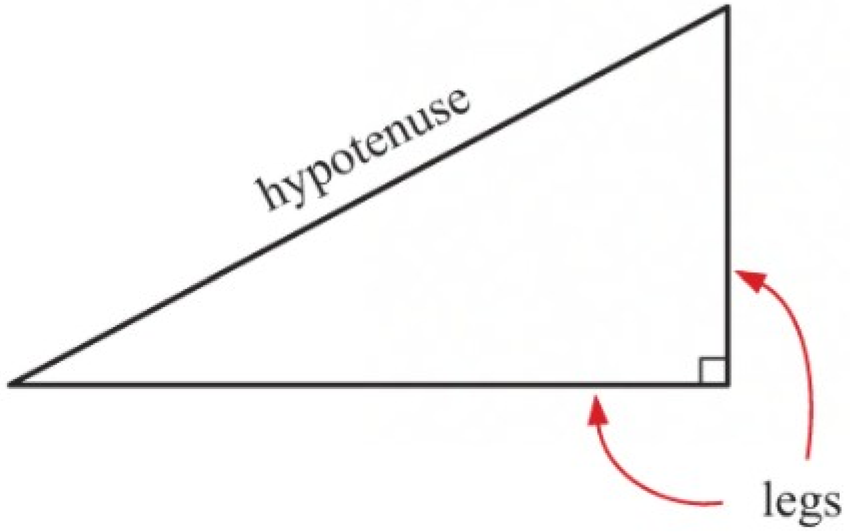
A PYTHAGORAS' THEOREM

A **right angled triangle** is a triangle which has a right angle as one of its angles.

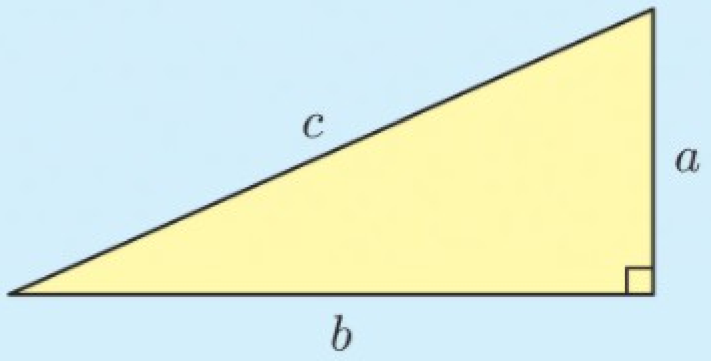
The side opposite the right angle is called the **hypotenuse**, and is the **longest** side of the triangle.

The other two sides are called the **legs** of the triangle.

Around 500 BC, the Greek mathematician **Pythagoras** discovered a rule which connects the lengths of the sides of right angled triangles.



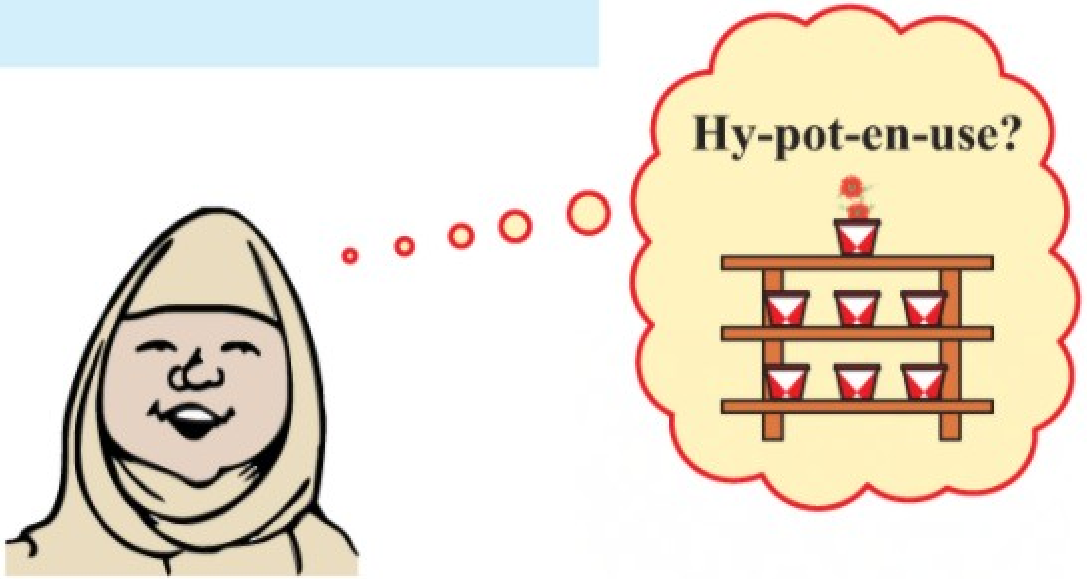
PYTHAGORAS' THEOREM

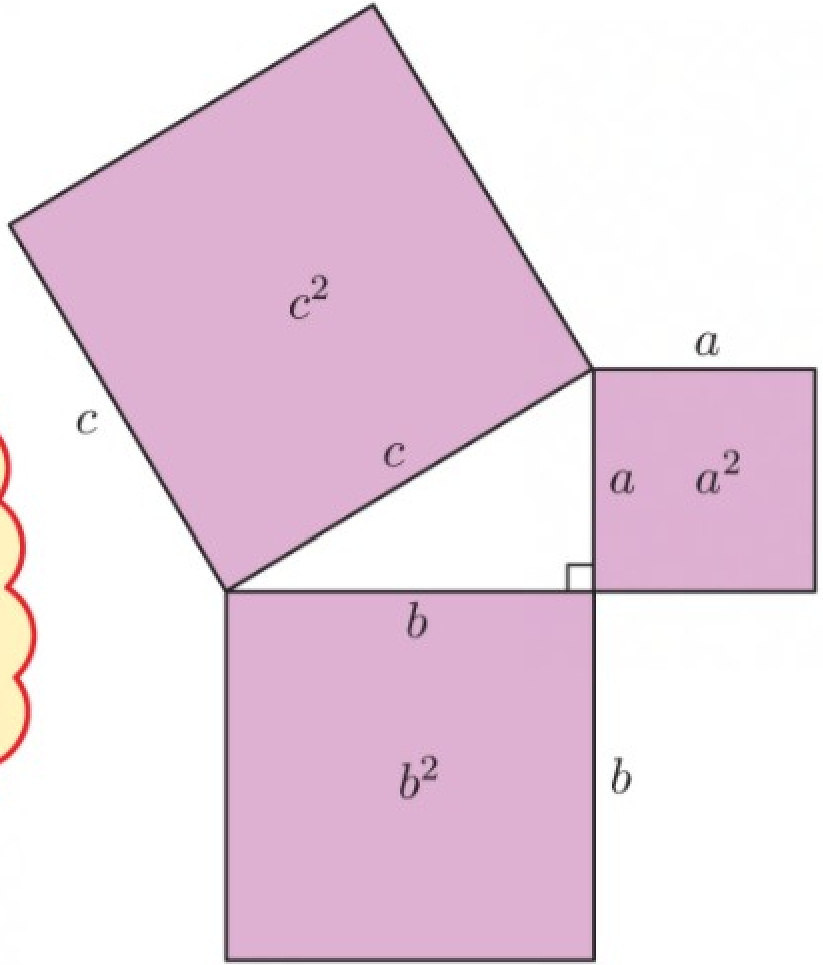


In a right angled triangle with hypotenuse c and legs a and b, $c^2 = a^2 + b^2$.

In geometric form, **Pythagoras' theorem** states that:

> In any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.


Hy-pot-en-use?

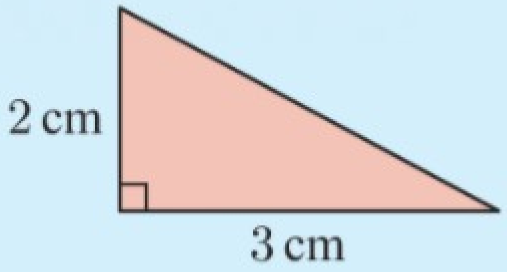


We can use Pythagoras' theorem to find unknown side lengths in right angled triangles.

Example 1

Self Tutor

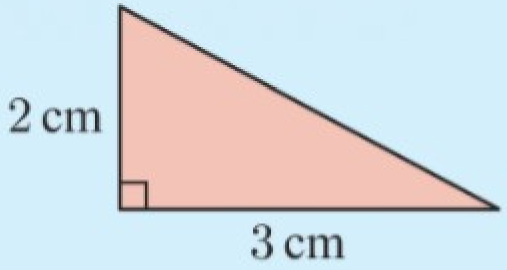
Find the length of the hypotenuse in:



Example 1

Self Tutor

Find the length of the hypotenuse in:



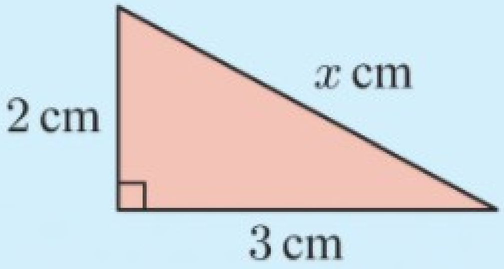
Let the hypotenuse have length x cm.

$\therefore\ x^2 = 3^2 + 2^2$ {Pythagoras}

$\therefore\ x^2 = 9 + 4$

$\therefore\ x^2 = 13$

$\therefore\ x = \sqrt{13}$ {as $x > 0$}

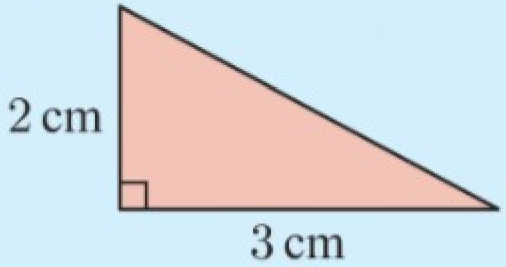


The hypotenuse is $\sqrt{13}$ cm long.

Example 1

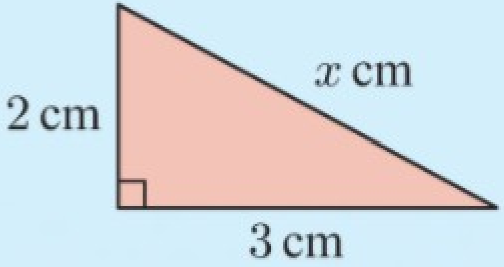
Self Tutor

Find the length of the hypotenuse in:



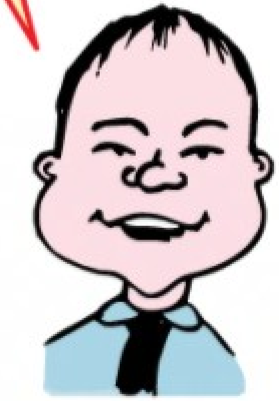
Let the hypotenuse have length x cm.

$$\therefore \quad x^2 = 3^2 + 2^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad x^2 = 9 + 4$$
$$\therefore \quad x^2 = 13$$
$$\therefore \quad x = \sqrt{13} \qquad \{\text{as } x > 0\}$$



The hypotenuse is $\sqrt{13}$ cm long.

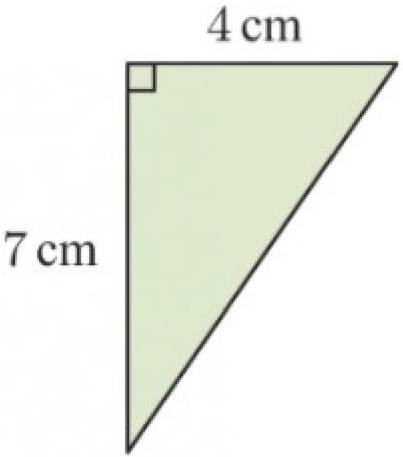
If $x^2 = k$, then $x = \pm\sqrt{k}$, but we reject $-\sqrt{k}$ as lengths must be positive!



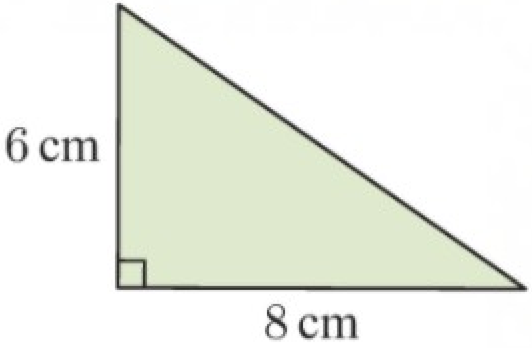
EXERCISE 5A

1 Find the length of the hypotenuse of each of the following triangles, leaving your answer in simplest radical form where appropriate:

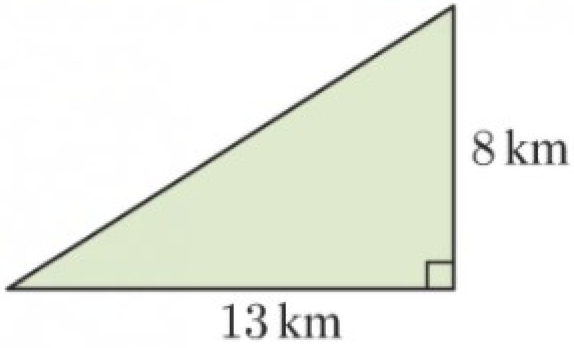
a



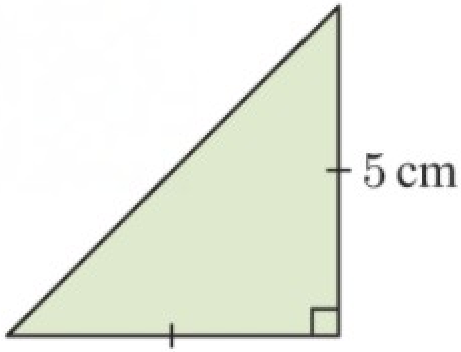
b



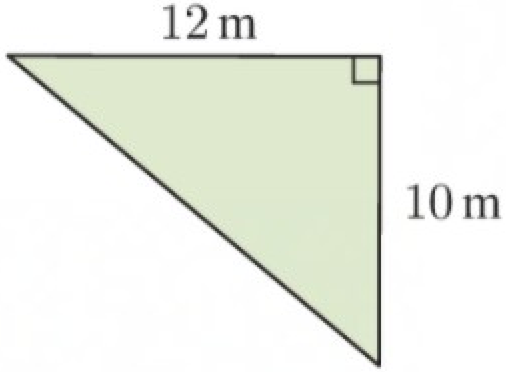
c



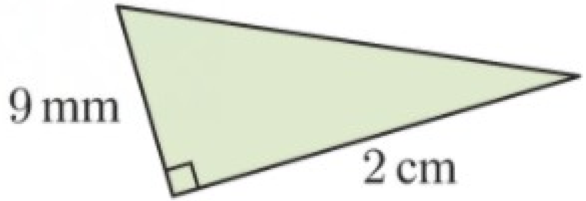
d



e

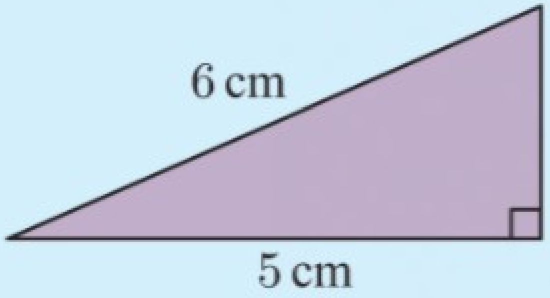


f



Example 2 **Self Tutor**

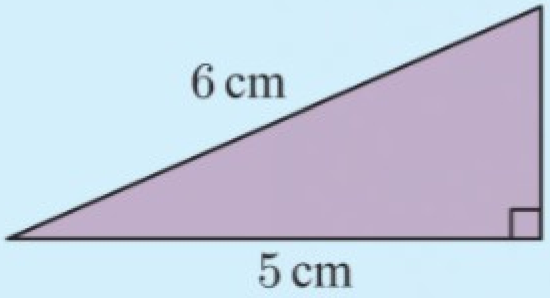
Find the length of the third side of the triangle.



Example 2

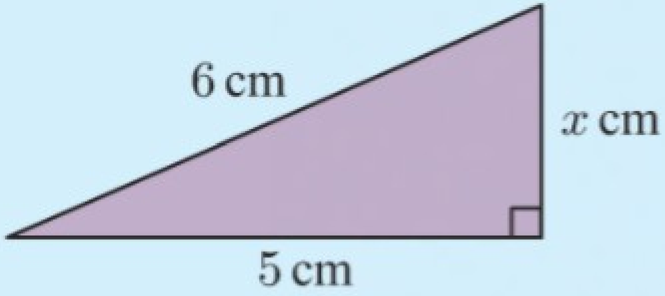
Self Tutor

Find the length of the third side of the triangle.



Let the third side have length x cm.

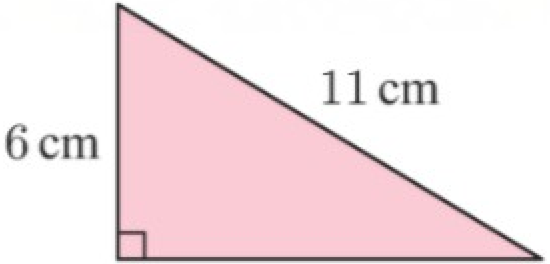
$$\begin{aligned}\therefore\quad x^2+5^2&=6^2 &&\{\text{Pythagoras}\}\\ \therefore\quad x^2+25&=36\\ \therefore\quad x^2&=11\\ \therefore\quad x&=\sqrt{11} &&\{\text{as } x>0\}\end{aligned}$$



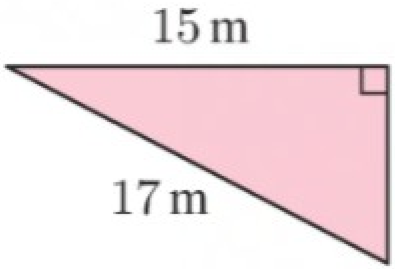
The third side is $\sqrt{11}$ cm long.

2 Find the length of the third side of each of the following right angled triangles. Where appropriate, leave your answer in simplest radical form.

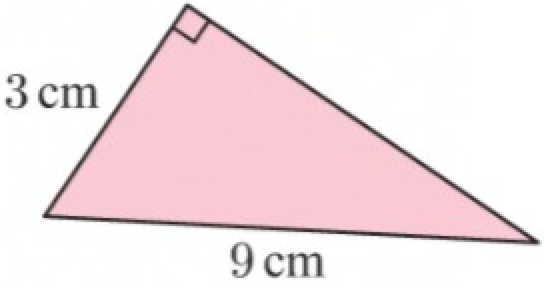
a



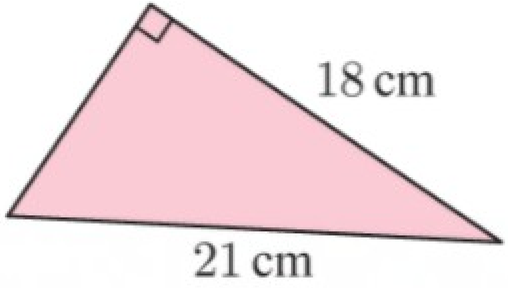
b



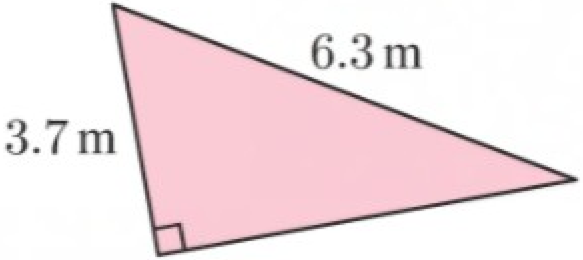
c



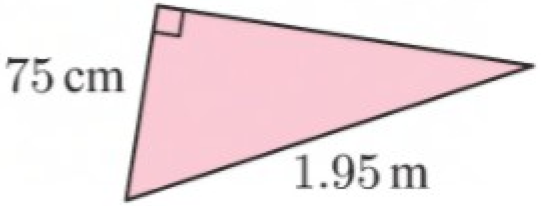
d



e

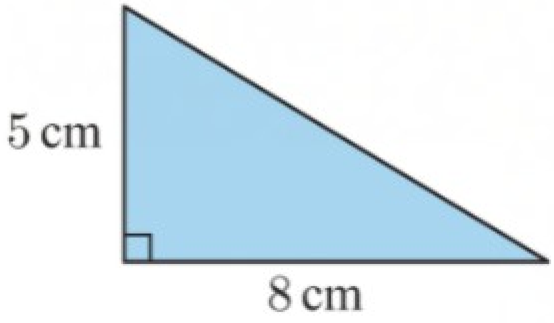


f

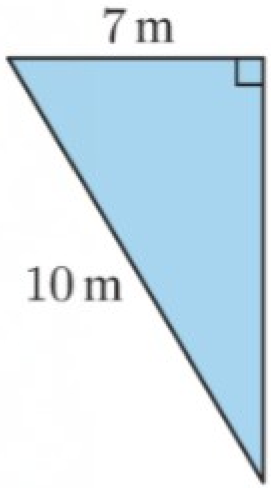


3 Find the length of the unknown side of each of the following right angled triangles. Give your answer to 1 decimal place where appropriate.

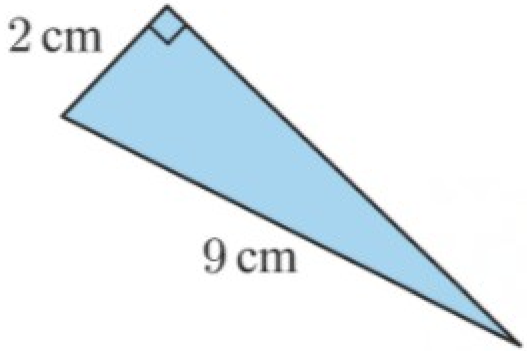
a



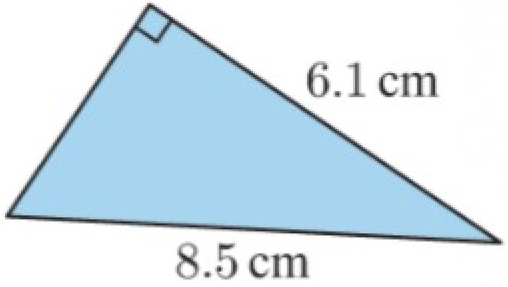
b



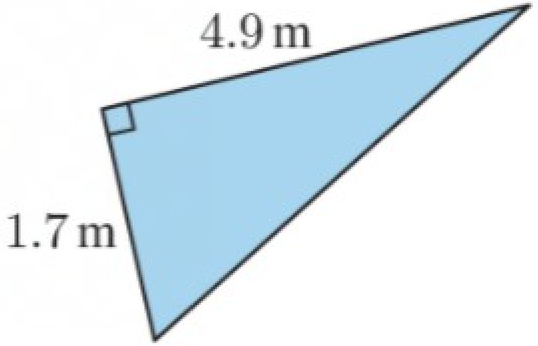
c



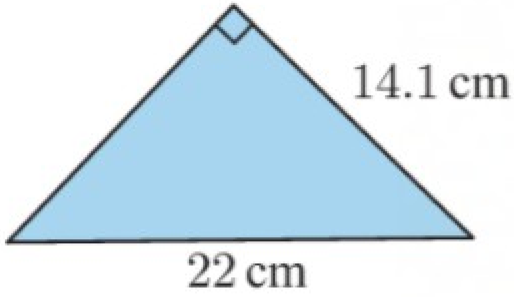
d



e

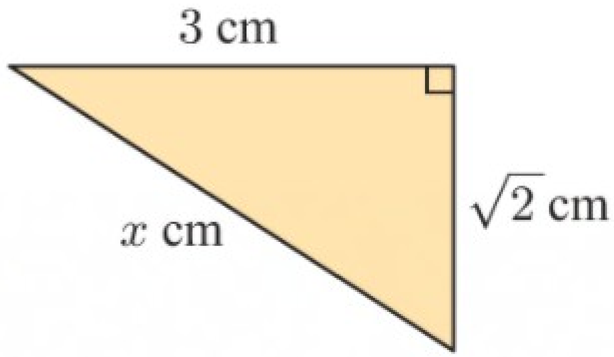


f

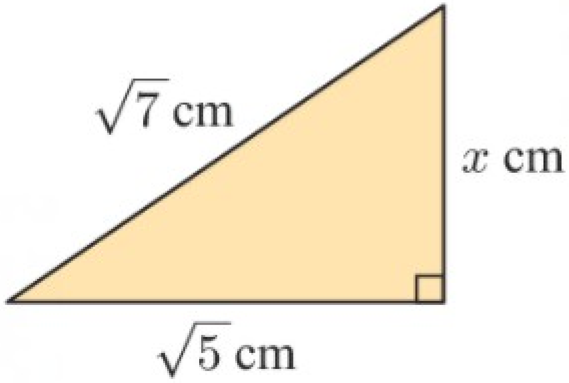


4 Find x in each of the following:

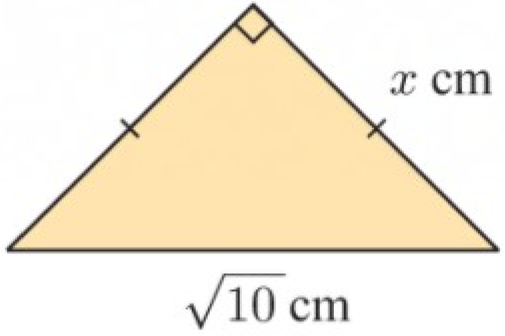
a



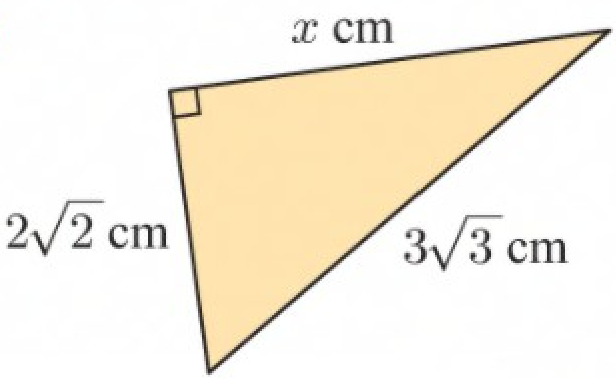
b



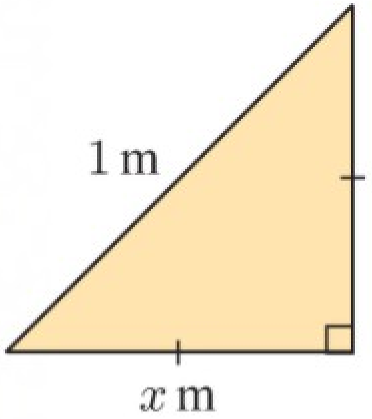
c



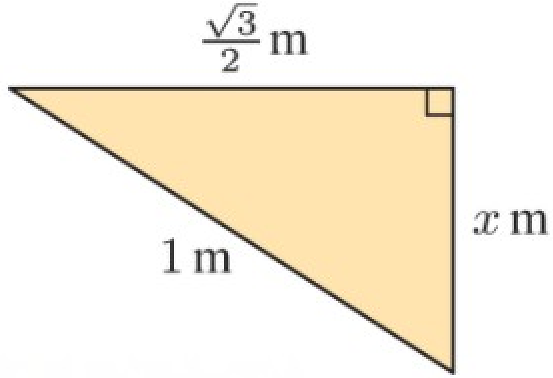
d



e

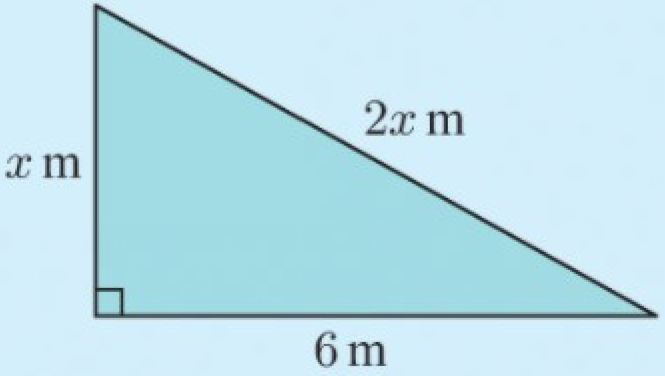


f



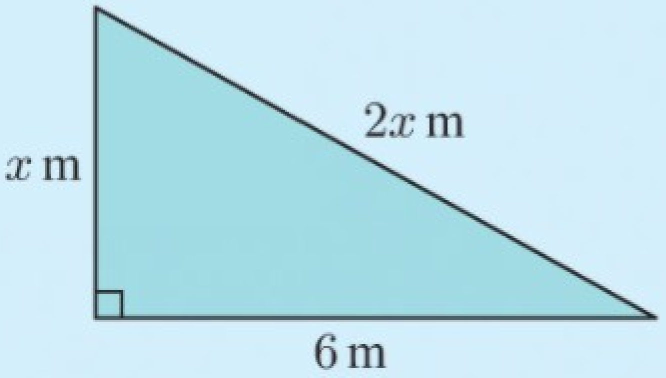
Example 3

Find x:



Example 3

Find x:

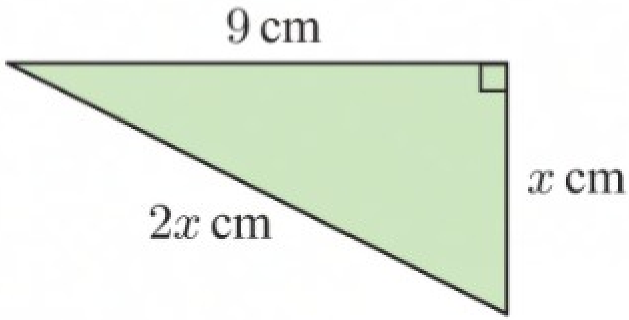


Self Tutor

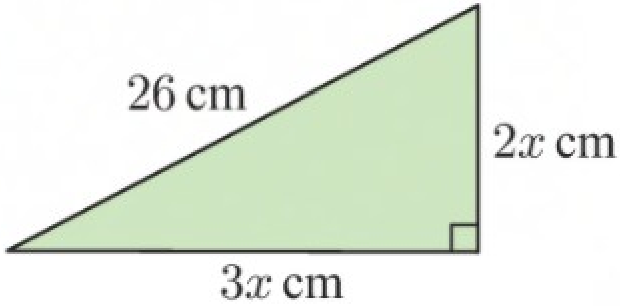
$$\begin{aligned} (2x)^2 &= x^2 + 6^2 && \{\text{Pythagoras}\} \\ \therefore\ 4x^2 &= x^2 + 36 \\ \therefore\ 3x^2 &= 36 \\ \therefore\ x^2 &= 12 \\ \therefore\ x &= \sqrt{12} && \{\text{as } x > 0\} \\ \therefore\ x &= 2\sqrt{3} \end{aligned}$$

5 Find the value of x:

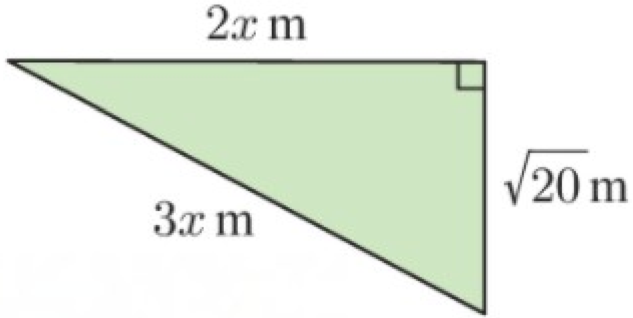
a



b

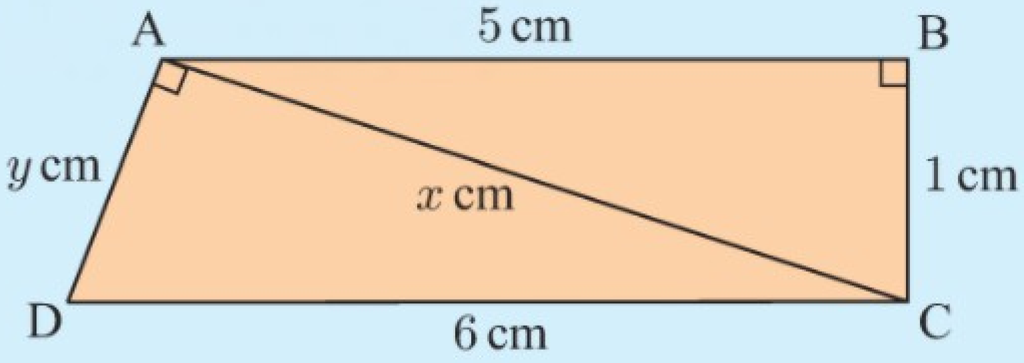


c



Example 4

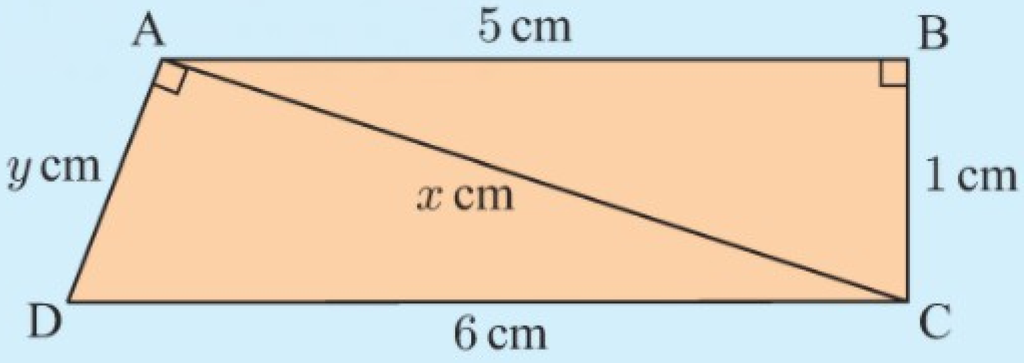
Find the values of the unknowns:



Example 4

Self Tutor

Find the values of the unknowns:



In triangle ABC, the hypotenuse is x cm.

$\therefore \quad x^2 = 5^2 + 1^2 \qquad$ {Pythagoras}

$\therefore \quad x^2 = 26$

$\therefore \quad x = \sqrt{26} \qquad$ {as $x > 0$}

In triangle ACD, the hypotenuse is 6 cm.

$\therefore \quad y^2 + (\sqrt{26})^2 = 6^2 \qquad$ {Pythagoras}

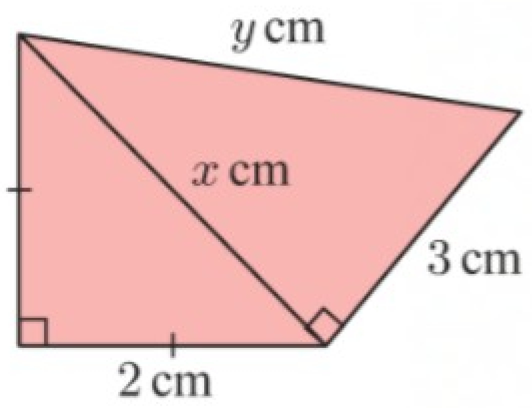
$\therefore \quad y^2 + 26 = 36$

$\therefore \quad y^2 = 10$

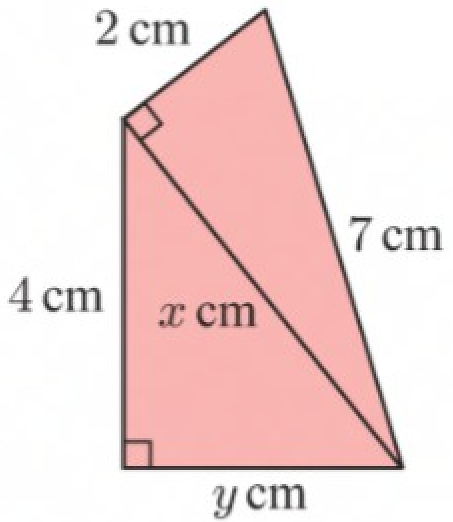
$\therefore \quad y = \sqrt{10} \qquad$ {as $y > 0$}

6 Find the values of the unknowns:

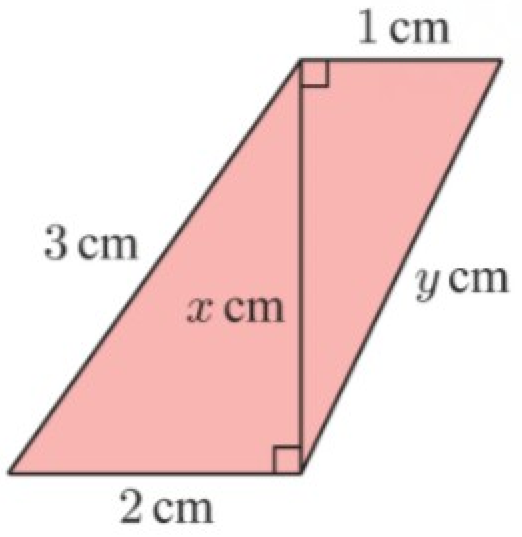
a



b

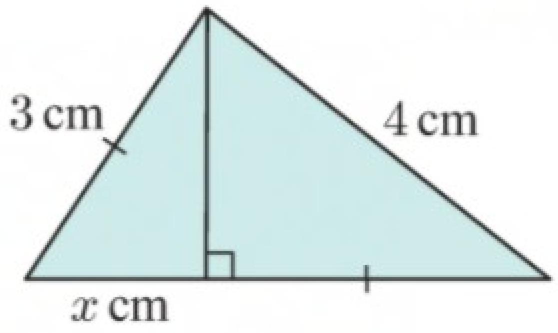


c

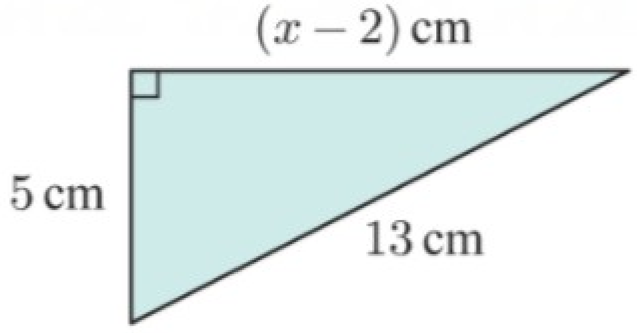


7 Find x:

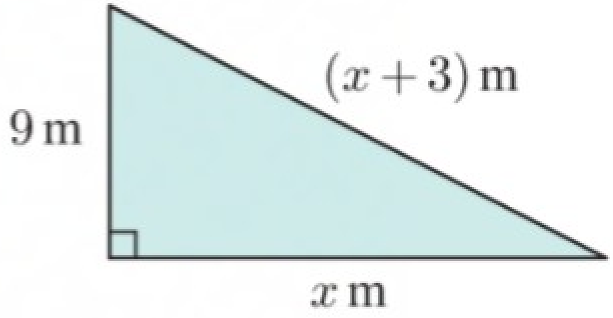
a



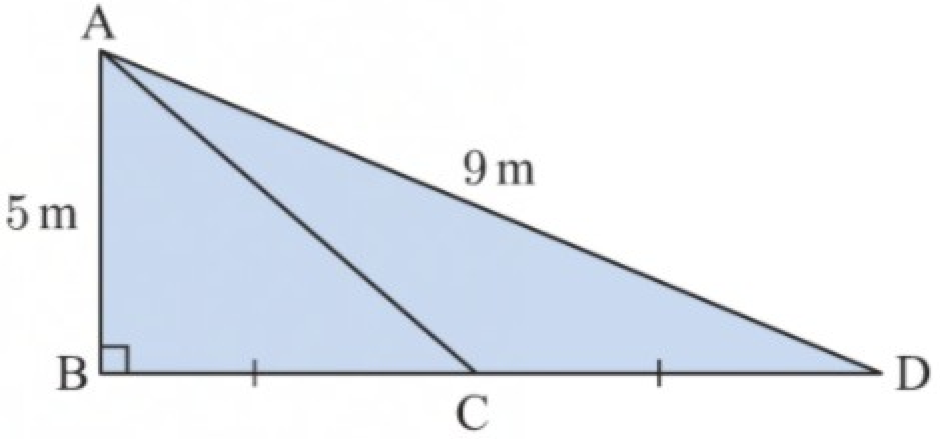
b



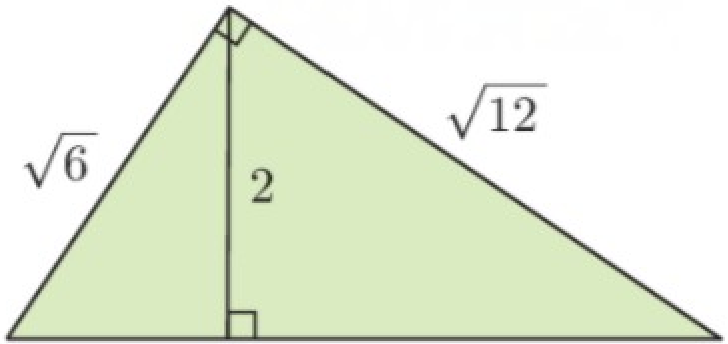
c



8 Find the length of [AC]:

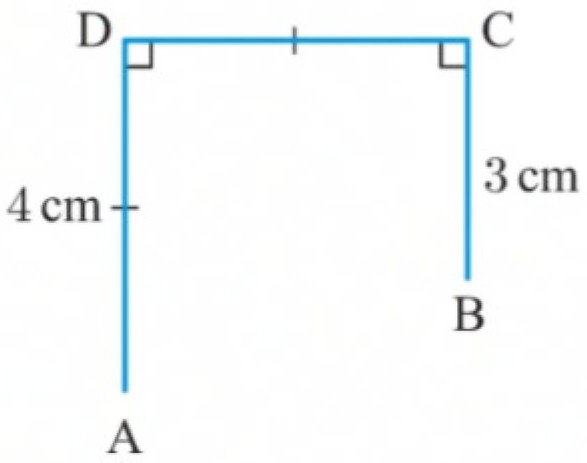


9 Use the figure below to show that $\sqrt{2}+\sqrt{8}=\sqrt{18}$.

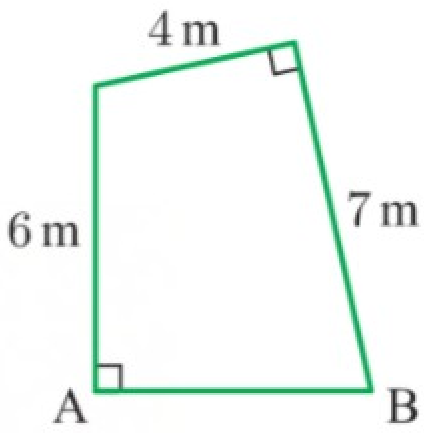


10 Find the distance AB in each of the following figures:

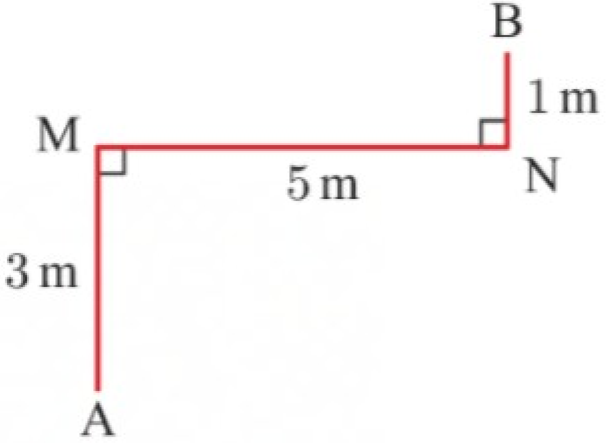
a



b



c



EXERCISE 5A

1 **a** $\sqrt{65}$ cm **b** 10 cm **c** $\sqrt{233}$ km **d** $5\sqrt{2}$ cm
e $2\sqrt{61}$ m **f** $\sqrt{481}$ mm

2 **a** $\sqrt{85}$ cm **b** 8 m **c** $6\sqrt{2}$ cm **d** $3\sqrt{13}$ cm
e $\sqrt{26}$ m **f** 180 cm or 1.80 m

3 **a** ≈ 9.4 cm **b** ≈ 7.1 m **c** ≈ 8.8 cm **d** ≈ 5.9 cm
e ≈ 5.2 m **f** ≈ 16.9 cm

4 **a** $x = \sqrt{11}$ **b** $x = \sqrt{2}$ **c** $x = \sqrt{5}$ **d** $x = \sqrt{19}$
e $x = \frac{1}{\sqrt{2}}$ **f** $x = \frac{1}{2}$

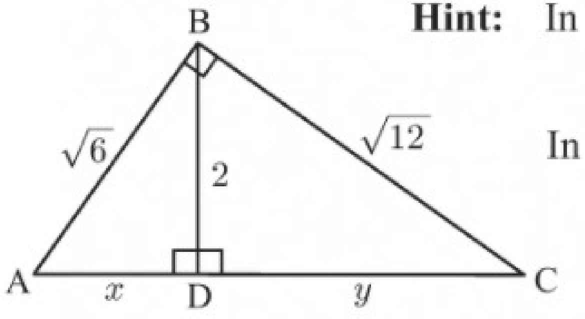
5 **a** $x = 3\sqrt{3}$ **b** $x = 2\sqrt{13}$ **c** $x = 2$

6 **a** $x = 2\sqrt{2}, \quad y = \sqrt{17}$ **b** $x = 3\sqrt{5}, \quad y = \sqrt{29}$
c $x = \sqrt{5}, \quad y = \sqrt{6}$

7 **a** $x = \sqrt{2}$ **b** $x = 14$ **c** $x = 12$

8 $\text{AC} = \sqrt{39}$ m ≈ 6.24 m

9



Hint: In $\triangle$ABD, $x^2 = 6 - 2^2$
$\therefore \quad x = \sqrt{2}, \quad x > 0$

In $\triangle$BCD,
$y^2 + 2^2 = 12$
$\therefore \quad y = \sqrt{8}, \quad y > 0$

In $\triangle$ABC, $\text{AC}^2 = (\sqrt{6})^2 + (\sqrt{12})^2 = 18 \quad \therefore \text{AC} = \sqrt{18}$

10 **a** $\sqrt{17}$ cm **b** $\sqrt{29}$ m **c** $\sqrt{41}$ m